Multi-Criteria Fuel Distribution: A Case Study

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Abstract. In this paper a multi-criteria fuel distribution problem similar to the common CVRP is considered with the real-life coordinates of gas stations obtained from a certain petrol company. The optimized criteria are the total distance of all tours and the number of tours. A certain method of solution representation, which ensures a feasible solution was used along a decoding scheme. A simulated annealing (SA) metaheuristic algorithm was implemented in order to obtain the approximations of the Pareto set for a number of instances with varying number of gas stations and demands for each station. The results show 17–18% improvement over the starting solution in all cases for both criteria. Moreover, the number of found solutions per instance increases with the number of gas stations. The improvements can be translated directly into profit, which was shown as well.

Keywords: Fuel distribution · CVRP · Simulated annealing · Multi-criteria optimization · Case study

1 Introduction

Vehicle routing problems (VRP), which aim at servicing demands of a given set of customers with a fleet of vehicles are one of the more common and more complex problems in combinatorial optimization and have great practical applications in the field of transportation and logistics. Moreover, they are characterized by their significant complexity, in particular many variants of they VRP are consider NP-complete/NP-hard and for VRP the problem sizes considered solvable are two orders of magnitude lower than compared to the less general Travelling Salesman Problem (TSP) as estimated by Ralphs *et. al* [11].

VRP problems often include additional constraints like limited capacity for vehicles (CVRP), limited distance per vehicle/tour (DVRP) or dedicated time windows when the client can be serviced (VRP-TW). Mixed variants (DCVRP, CVRP-TW *etc.*) also exist. The goal for common single-criteria VRP is the minimization of such criteria like the total distance, the maximum distance or the maximization of the vehicle utilization (*i.e.* average or minimal loading volume used). Bi- and multi-criteria approaches to VRP are less common.

Fuel distribution is one example of the vehicle routing problems encountered in practice, which has considerable impact on the various aspect of daily life

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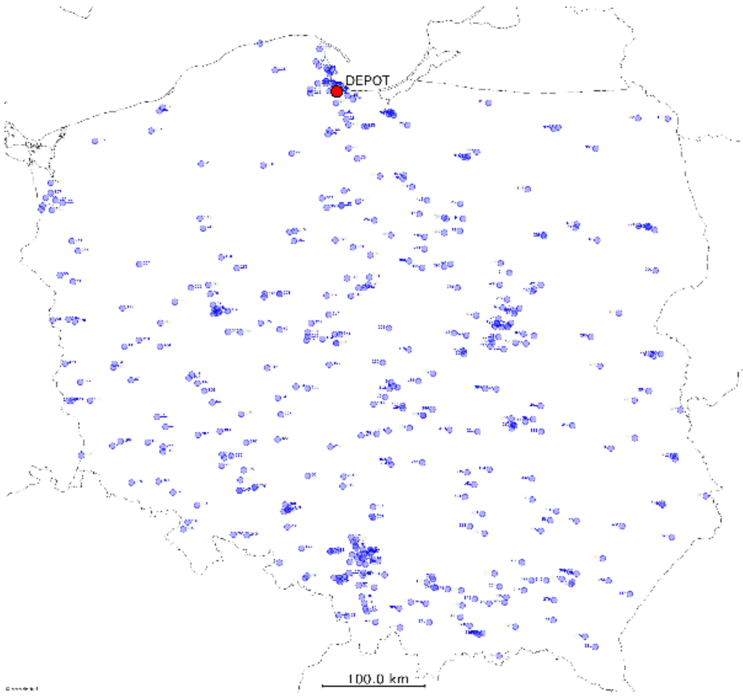

Fig. 1. Gas stations and refinery for considered fuel distribution problem

around the globe, as access to fuels determines developments of many sectors of the economy. In this paper we focus our efforts on a specific instance of the fuel distribution problem. Namely, we consider a real-life petrol company with a network of 424 gas stations and one refinery acting as a depot, similarly to the usual VRP. Fig. 1 shows locations of the refinery and all gas stations, taken from real-world data. The distances between each pair of locations were computed using Google Maps API, resulting in distance matrix numbering 425^2 (or over 180625) edges. Let us also take note that the depot is located off the center of the map. Moreover, the analysis of some edges shows that the graph is directed *i.e.* distance from location A to B is not necessarily the same as distance from B to A. According to aforementioned estimate, VRP with 425 nodes is as difficult to solve as TSP with 42 500 nodes.

In this paper we decided to use multi-criteria approach to the fuel distribution problem, as we consider it more practical and closer to the real-life needs of such companies. The chosen criteria are the total distance of all tours and the number of tours (the latter can be translated into the numbers of tankers). The remainder of the paper is organized as follows. In Section 2 we present a brief literature overview concerning the multi-criteria vehicle routing problems. Section 3 contains the definition of the problem, while in section 4 we describe the chosen solution representation and problem solving method. Section 5 shows the results of the computer experiment and, finally, section 7 offers the conclusions.

2 Literature Overview

Over the last decade multi-criteria transportation problems have received some interest. Albeit, most of the multi-criteria VRP approaches were based on evolutionary algorithms (EA), while only some researchers of local search methods like simulated annealing (SA) or tabu search (TS). We will mention a few of all those approaches in the following section.

Bowerman *et al.* proposed one of the promising scalar techniques in [1]. In the evaluation, they used five different sets of weights chosen by a decision-maker. Proposed algorithm first groups the nodes into clusters that can be served by a vehicle, and then determines a tour. It's an allocation-routing-location strategy. Lee and Ueng proposed insertion algorithm in [7]. In each iteration, using a saving criterion, it adds one node to the vehicle with the shortest path. Other insertion heuristic was proposed by Zografos and Androutsopoulos in [15]. The selection of the customers to be inserted allows both routed and unrouted demand points to be inserted.

In [8] Pacheco and Marti optimize the makespan objective for every possible value of the second objective and then use a TS algorithm to solve each problem. Their approach uses a scalar method, called the ϵ-constraint. Similar strategy was used by Corberan *et al.* [3], albeit authors used scatter search approach instead of TS.

The Pareto concept was frequently used within an evolutionary multi-objective framework. A modification of NSGA-II algorithm was proposed in [12]. Proposed algorithm uses local search method in each iteration, in order to further improve the Pareto frontier approximation. Pareto dominance has also been used by Ulungu *et al.* in a SA technique called Multi-Objective Simulated Annealing (MOSA) [14]. Paquete et al. [9] have called upon Pareto Local Search techniques. These techniques are based on the principle that the next current solution is chosen from the non-dominated solutions of the neighborhood. Rudy and Żelazny proposed a new hybrid of genetic algorithm and ant colony optimization in [13]. Tests shown it performed well in multi-objective job shop scheduling problem.

Some studies employ neither scalar nor Pareto methods to solve multi-objective routing problems. These non-scalar and non-Pareto methods are based on lexicographic strategies or specific heuristics. Aforementioned VEGA algorithm might be included, as an example of this specific heuristics. While lexicographic strategy was used in works of Keller and Goodchild [5,6]. Their approach was such, that the objectives are each assigned a priority value, and the problems are solved in order of decreasing priority. When one objective has been optimized, its value cannot be changed and it becomes a new constraint for the problem.

Moreover, Żelazny *et al.* proposed a new concept of solving VRP in [4]. New approach to GPU implementation of TS algorithm was proposed and significant speed-ups, as well as an improvement of the approximation of the Pareto front were observed.

3 Problem Description

The problem is defined as a modification of the basic CVRP problem, which is considered NP-hard. There is given a directed graph $G = (N, E)$ with $|N| - 1$ vertices representing filling stations and one vertex B representing the depot (in our case the oil refinery). There is an edge between each pair of vertices $n_1, n_2 \in N$, thus the number of edges is $|N|^2 - |N|$. The directed edge $e(n_1, n_2)$ from vertex n_1 to n_2 represents the driving distance from gas station v_1 to n_2. In general $(n_1, n_2) \neq (n_2, n_1)$. Moreover, each station has a demand for two fuel types: diesel fuel (henceforth called diesel) and petrol, both expressed in the number of liters. The demands are met by a set of tank trucks (henceforth called tankers), with each tanker outfitted with four separate fuel compartment. We assume that compartment can contain the fuel of only one type (either diesel or gasoline) or be empty. Thus, each tanker has a limited capacity $C = V \times K$, where V is the volume of a single compartment and K is the number of compartments per tanker (we assume that compartments are identical).

The goal is to find a solution that minimizes our bi-criteria goal function. The considered criteria are: 1) the total number of routes R and 2) the total distance of all routes D given as:

$$D = \sum_{i=1}^{R} \sum_{n \in N_i} w(n), \tag{1}$$

where R is the total number of routes, N_i is the set of edges in route i and $w(n)$ is the weight of edge n.

$$s^* = \min_{s \in \mathbb{S}_{\text{feas}}} f(D_s, R_s), \tag{2}$$

where D_s (R_s) is the total distance (number of routes) for the solution s, f is our goal function and $\mathbb{S}_{\text{feas}}$ is the set of feasible solutions. A solution is feasible if:

- the demands for all gas stations are met,
- all routes begin and end in the depot B,
- the sum of fuel transported on any single route does not exceed C.

4 Model and Solving Method

In this chapter we will present the solution representation as well as decoding method used to calculate the values of the goal function. We will also describe the chosen metaheuristic algorithm.

4.1 Representation

Let us assume N gas stations and K identical compartments per tanker. For every station i we are given two numbers d_i and p_i indicating diesel and petrol demands respectively. The required number of compartments Z_i for station i is then calculated as:

$$Z_i = \left\lceil \frac{d_i}{V} \right\rceil + \left\lceil \frac{p_i}{V} \right\rceil, \tag{3}$$

where V is the volume of a single compartment. In the next step a sequence of numbers is created. For each city i its number is written Z_i times. For example, for 3 cities with $d_1 = 4000$, $d_2 = 1500$, $d_3 = 2100$, $p_1 = 2200$, $p_2 = 3800$, $p_3 = 4700$ and $V = 5000$ (all in liters) we have:

$$Z_1 = 4 + 3 = 7, \tag{4}$$

$$Z_2 = 2 + 4 = 6, \tag{5}$$

$$Z_3 = 3 + 5 = 8, \tag{6}$$

and the resulting sequence is:

$$1\,1\,1\,1\,1\,1\,1\,2\,2\,2\,2\,2\,2\,3\,3\,3\,3\,3\,3\,3\,3. \tag{7}$$

Next each station is filled with zeroes to the nearest multiple of K (if Z_i is divisible by K then no zeroes are added). In our example (we assume $K = 4$) we have:

$$|1\,1\,1\,1|1\,1\,1\,0|2\,2\,2\,2|2\,2\,0\,0|3\,3\,3\,3|3\,3\,3\,3|. \tag{8}$$

We thus obtained a representation of 6 "tours" by placing additional zeroes. Let us call this an extended representation. This representation will be further modified due to the neighborhood search during the course of our metaheuristic algorithm. For example, the above representation can be modified into the following:

$$|1\,1\,2\,3|2\,3\,3\,3|1\,2\,3\,2|1\,0\,3\,3|1\,3\,0\,0|1\,1\,2\,2|. \tag{9}$$

However, before the metaheuristic algorithm starts we add one more step. Let us consider all tours consisting of only one station *e.g.* $|2\,2\,2\,2|$. Those tours mean that tanker leaves from the refinery (location 0), then delivers fuel (all its four compartments) to specific station (2 in this example) and then goes back to the refinery. Such tours are removed from the representation. However, they still affect the values of the goal function. In our example from representation (8) four tours are removed and we get the following representation:

$$|1\,1\,1\,0|2\,2\,0\,0|. \tag{10}$$

The next essential part of the algorithm is the decoding procedure responsible for transforming a given representation into an actual tour and calculating the value of the goal function. Let us consider the following representation ($K = 4$):

$$|1\,2\,3\,3|5\,6\,7\,0|. \tag{11}$$

The decoding procedure works by employing the nearest neighbor method. In our example the first tour consists of stations 1, 2 and 3. Thus, we first visit the station with the shortest distance from the current location (*i.e.* from refinery at location 0). Let us assume this is station 2. Next, we repeat the reasoning, searching for the closest station to the current location (station 2). Let it be station 3. The last station to visit is station 1 and then we go back to the refinery. The second tour consists of stations 5, 6 and 7 (0 means empty compartment). We use the nearest neighbor method again to chose the order of stations to visit. In result, both tours might look as follows:

$$0 \rightarrow 2 \rightarrow 3 \rightarrow 3 \rightarrow 1 \rightarrow 0 \tag{12}$$

$$0 \rightarrow 6 \rightarrow 7 \rightarrow 5 \rightarrow 0 \tag{13}$$

Now that we know the number of tours we can compute the value of the goal function. Let us also note that tours consisting of only zeroes (*i.e.* $|0\,0\,0\,0|$) are considered empty and do not affect the value of the goal function.

Let us also note some of the properties of the given representation. First, the lower bound for the number of resulting tours is given as:

$$\frac{\sum_{i=1}^{N} Z_i}{K}, \tag{14}$$

while the upper bound is given as $\frac{Z^*}{K}$, where Z^* is the number of compartments (including empty ones) in the extended representation. Moreover, the solutions resulting from the chosen representation are always feasible, as the number of compartments for each station is always the same as the demand of the given station and no tour has a load exceeding the assumed capacity C.

4.2 Simulated Annealing

Classic SA was formulated for the case of optimization of a single objective function $f(x)$. Proposed algorithm, although similar to original version, was adapted to solving multi-objective problems. Moreover, we have added an auto-tuning phase, in which parameters of the algorithm are set.

In each iteration, from the neighborhood $N(x)$ of current solution x, a neighbor x' is selected. The neighbor is chosen randomly, assuming uniform distribution of probability. First, we use Pareto-optimality concept to determine whether solution x' dominates current solution x and replaces it in next iteration of the algorithm. In other case, when solution x' is dominated by x or neither solution dominates the other, we evaluate the objective difference $\Delta = \sqrt{\sum_{i=1}^{q}(f_i(x') - f_i(x))^2}$. The value Δ is no less than $\max_{1 \le i \le q}\{f_i(x') - f_i(x)\}$ and each difference $f_i(x') - f_i(x)$, $i = 1, ..., q$ influence this value i.e. increase it. Solution x' is accepted as the new solution for the next iteration with probability $p = exp(-\Delta/T)$, where T is parameter called temperature. Starting from the initial temperature T_0, the temperature is reduced slowly with each iteration using cooling scheme. Higher values of temperature T give a higher probability of acceptance. The influence of quality of solution (measured by Δ) on acceptance probability is opposite. We used a geometric cooling scheme, in which the temperature during s-th iteration is $T_s = \lambda T_0$, where λ is a parameter.

Similar algorithm has been successfully used in [10]. A parallel SA algorithm was proposed in [2]. Super linear speed-up was obtained when using representative-based neighborhood.

5 Computer Experiment

Proposed algorithm was implemented in C++ programming language and compiled with Embarcadero C++Builder XE7 Professional. The program was tested on Intel Core i7-3770 3400MHz, 8GB RAM and running Windows 7 64-bit.

We have tested our SA algorithm using real-world data obtained from a certain petrol company and accordingly generated orders. For each test instance and for each run of algorithms, we collected the following values:

- $\delta F1$ – improvement of first optimization criteria over initial solution,
- $\delta F2$ – improvement of second optimization criteria over initial solution,
- $|P|$ – average number of non-dominated solutions in Pareto front.

Tab. 1 shows summary of results obtained for all problem instances considered in this paper. The mean improvement was over 18% for the first criterion, and over 17% for the second. Moreover, the average number of non-dominated solutions increased with the size of the problem. It is important to note that the number of nodes indicates only the number of gas stations (*e.g.* the size of the distance matrix), while the true problem size depends on the size of the demands for each station.

Table 1. The evaluation of the solutions – summary

Nodes	$\delta F1$ [%]	$\delta F2$ [%]	$\|P\|$
25	18.07	15.83	1.1
50	18.66	17.02	1.8
75	18.33	17.19	2.1
100	18.57	17.25	1.8
150	18.45	17.52	2.7
200	18.13	17.50	2.6
275	18.35	17.40	3.7
350	18.33	17.45	3.6
425	18.27	17.65	5.0
Average	18.35	17.20	2.71

Proposed SA has improved the initial solution significantly. It is worth mentioning, that proposed method was extremely fast. Biggest instances were computed in less than 5 minutes, while smallest took around 1 second. In comparison, using other decoding scheme took over 30 minutes to compute instances with 425 nodes and provided worse solutions.

Table 2. The evaluation of biggest instances

Nodes	Instance No.	$\delta F1$ [%]	$\delta F2$ [%]	$\|P\|$	Nodes	Instance No.	$\delta F1$ [%]	$\delta F2$ [%]	$\|P\|$
200	1	17.74	17.29	4	350	1	18.03	17.65	2
	2	19.75	19.60	3		2	18.94	17.97	4
	3	19.03	17.78	1		3	17.34	16.29	6
	4	18.71	18.83	2		4	19.53	18.43	3
	5	17.75	16.84	4		5	18.90	18.43	2
	6	17.06	16.45	4		6	17.54	16.42	1
	7	17.85	17.36	2		7	19.28	18.12	3
	8	17.79	16.12	4		8	18.82	17.88	4
	9	17.99	16.98	1		9	17.67	16.80	6
	10	17.66	17.72	1		10	17.24	16.49	5
275	1	18.65	17.02	3	424	1	18.03	17.36	4
	2	18.75	17.93	3		2	19.33	18.53	1
	3	18.44	16.72	4		3	17.76	17.36	4
	4	18.71	18.32	3		4	17.47	16.78	9
	5	18.25	16.76	2		5	17.39	17.54	7
	6	18.21	17.19	4		6	18.20	17.83	4
	7	18.25	17.84	4		7	17.37	16.50	3
	8	17.92	17.61	6		8	19.77	18.47	5
	9	16.91	15.89	6		9	19.15	18.30	7
	10	19.36	18.71	2		10	18.24	17.80	6

We also provided, in Tab. 2, more detailed results for the 4 largest instance sizes: 200, 275, 350 and 424 gas stations respectively (instances of smaller size were omitted for the sake of brevity). The results serve to show that improvement rates for all considered instances remain close to the 17% on both criteria. In fact, all improvement values from Tab. 2 range between 15.89% and 19.57%.

Let us also note the practical significance of such optimization. For example, SA algorithm for one of the instances for 424 stations (depot is not considered a gas station) managed to reduce the total distance (in km) from 694194 to 568694. Assuming that a tanker consumes 30 liters of fuel per 100 kilometers (real values are even higher if driving style is taken into account) and that liter of fuel costs around 1.2 € (European countries), then the total fuel costs of such distribution operation can be reduced by around 45000 € or over 55000 $.

6 Conclusions

In this paper we considered a multi-criteria fuel distribution problem similar to the common CVRP. The chosen criteria were the total distance of all tours and the number of all tours, without explicitly stating the available number of tankers. The distance matrix was created based on the real-life data of a certain petrol company with refinery acting as a depot.

90 instances of the problem (*i.e.* fuel demands for stations) were defined and combined into group of ten, depending on the number of stations. We employed a simple solution representation method, which provides only feasible solutions and then applied it to the SA metaheuristic algorithm to obtain the approximation of the Pareto set. Our research yielded good results (consistent 17–18% improvement of starting solution for both criteria) in reasonable time (5 minutes for largest instances with 424 stations).

Further research include consideration of other criteria (especially use of 3 criteria at once) and development of a Decision Support System (DSS) for the presented fuel distribution problem.

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